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Summary Sheet

DREAM: An Ecological Game to Grow and Migrate Three Dragons

Summary

In the fictional television series *Game of Thrones*, three dragons raised by *Daenerys Targaryen* are eye-catching for their gigantic size and tremendous power. An interesting hypothesis is proposed that what if three dragons are living today, while more details such as the dragons' characteristics, behavior, habits, diet, and interaction with their environment are still in need of description and calculation.

Aimed to give a picture of three dragons existing in reality, we devise ***Dragon-Raising Ecology and Annual Migration (DREAM)*** model to describe the biological features of dragons and calculate energy expenditures and ecological requirements. The life of a dragon can be split into the stage of living in the reserve and the stage of migration every year, so is ***DREAM*** model into ***Dragon-Raising Ecology*** model and ***Annual Migration*** model.

In ***Dragon-Raising Ecology*** model, **Logistic Model**, **Fourier Law of Heat Conduction**, **Maxwell's Velocity Distribution Law**, and other well-proved theories are employed to construct the foundations of our model. Meanwhile, **Markov Chain** is hired to calculate the probability of dragons in resting, flying, fighting, and death state. Combining ***Dragon-Raising Ecology*** model with analogy method and modest estimation, we obtain energy expenditures of weight growth, basic metabolism, temperature maintaning and evaporation energy dissipation.

In ***Annual Migration*** model, we apply **Thin Airfoil Theory** to calculating energy expenditures of flying energy dissipation. Adopting the integrity of our ***DREAM*** model, we further calculate total energy expenditures, caloric intake as well as the size of area and community required to support dragons.

Moreover, in order to simulate the real world, a lot of relavant data is collected from websites for analysis. At last we conclude from our modeling and calculation that during migration, a larger area and more food are necessary in the arctic region while less resources are needed in an arid region.

Our ***DREAM*** model shows strong robustness in sensitivity analysis. In the meantime, it can be applied to animal raising industry especially for ancient huge animals revived by gene technology in the near future.

Keywords: Logistic Model, Fourier Law of Heat Conduction, Maxwell's Velocity Distribution Law, Thin Airfoil Theory, Markov Chain

DREAM: An Ecological Game to Grow and Migrate Three Dragons

Contents

1 Introduction

1.1 Problem Background

Being all the rage, the fictional television series *Game of Thrones* vividly pictures a magic world featured by thrilling plots of wars and conquest. One of the main characters, *Daenerys Targaryen*, also called the "Mother of Dragons", brings up three ferocious dragons and therefore fighting fearlessly to recapture her lost throne [1]. With great curiosity of such creatures, we would like to visualize how the three dragons live in the world today, which would also be of help in the realm of science fiction composing.

Fiction notwithstanding, the idea of raising three dragons in modern society requires down-to-earth efforts to make it closer to reality. Reasonable assumptions are made for the sake of proper simplification. Since our team is assigned to analyze dragon characteristics, behavior, habits, diet, and interactions with the environment, various biological and ecological factors are taken into consideration.

A dragon is a large, serpent-like legendary creature that appears in the folklore of many cultures around the world [2]. In *Game of Thrones* which is based on Western culture, the three dragons raised by *Daenerys Targaryen* are two-legged, large-winged fiery animal. A portrait of this creature at birth in *Game of Thrones* is demonstrated in Figure 1. In the process of analyzing the problem of breeding dragons, we refer to realistic research of other animals. To a certain extent, the non-existent dragons could be analogous to dinosaurs that once lived on this planet, or other large animals with similar characteristics in one way or another.

Figure 1: A portrait of a two-legged, large-winged fiery dragon in the television series

Similar to dinosaur research, most aspects of biology cannot be observed directly but must be reconstructed by a variety of often speculative approaches [3]. In the analysis of the ecological impact and requirements, energy expenditures, caloric intake requirements, living conditions, as well as migration of the dragons, the strategy of mathematical modelling primely fulfills the requirements.

All in all, a combination of analogy method and mathematical modelling results in the solution of raising three dragons in modern society.

1.2 Our Work

To address the situation of raising three dragons in modern society, ***Dragon-Raising Ecology and Annual Migration (DREAM)*** model is proposed to depict the biological features of dragons and calculate their ecological impact and requirements. The model can be divided into two parts. ***Dragon-Raising Ecology*** model is adopted to estimate energy transfer of ecosystem with dragons. On the other hand, ***Annual Migration*** model is employed to predict the migration pattern of dragons and determine different resources required to support the three dragons in various regions. Figure 2 shows the comprehensive working flow of our ***DREAM*** model.

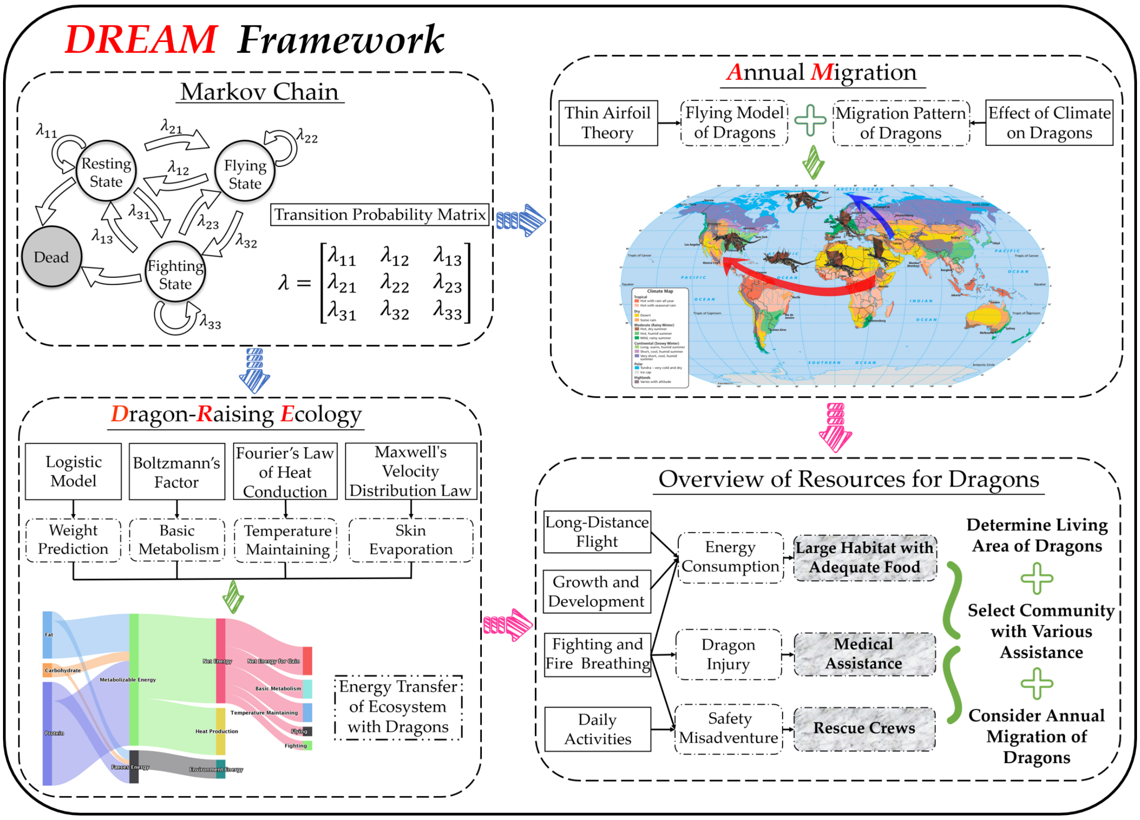


Figure 2: ***DREAM*** model to grow and migrate three dragons. **Markov Chain** depicts the state transition probability of the dragons. ***Dragon-Raising Ecology*** model and ***Annual Migration*** model respectively illustrate how to grow and migrate a dragon. With ***DREAM*** model defined, the resources required for dragons can be estimated based on various factors.

In Section 2.1, we state several basic assumptions. Section 2.2 contains the nomenclature used in model statement. Section 3 and Section 4 provide sufficient details about the ***Dragon-Raising Ecology*** model and ***Annual Migration*** model we develop. Section 5 carrys out the implementation of our ***DREAM*** model with specific data. Section 6 provides detailed strategies determining energy expenditures, living area, and community size required to support the dragons considering their migration and extends our model in some realistic applications. At last, we further study and analyse our models in Section 7 and make some conclusions in Section 8.

2 General Assumptions and Nomenclature

2.1 General Assumptions

Our model makes the following general assumptions:

1. Dragons are warm-blooded animal, able to fly great distance and breathe fire, with a maximum age of 220 years old, and a maximum weight of $130t$. These data from *A Song of Ice and Fire* is true of our dragons in reality.
2. Dragons are born in the reserve in warm temperate region, and will not migrate until they grow mature. A grown-up dragon embarks on a migration every year to arid and arctic region.
3. In the aspect of physiology, we mainly consider three facets: basic metabolism, temporature maintaining, and skin evaporation.
4. In terms of flight, the velocity of flying dragons is much lower than that of a traveling light. Meanwhile, dragon's wings are thin enough compared with its length, whose airfoil can be described by a parabola.

2.2 Nomenclature

In this paper we use the nomenclature in Table 1 to descibe our model. Other symbols that are used only once will be described later.

Table 1: Nomenclature

Symbol	Definition
λ_{ij}	Transition probability from state j to state i
$N(t)$	Relative number of total cells at a given time t
$\mu(r,t)$	Net proliferation rate of cells aged r at a given time t
B	Basic metabolic rate of dragons
M	Body mass of a dragon, which also means weight
T	Absolute temperature
B_0	Mass-normalized metabolic rate
L	Thickness of horny layer of dragons
$u(x,t)$	Temperature of location x in axis at a given time t
κ	Thermal conductivity, a measure of its ability to conduct heat
f	Wing camber of a dragon wing airfoil
c	Chord length of a dragon wing airfoil
L	Lift of a flying dragon
ρ	Air density
Γ	Circulation of the airfoil
l	Wing span of the dragon
γ	Attack angle
c_{air}	Coefficient of air resistance
S	Coronal section areas of the dragon
s	Relative humidity
v_e	Normalized evaporation velocity
Φ_{super}	Superficial heat transition flux density

3 Dragon-Raising Ecology

In this section, we focus on the research of dragon's energy expenditure when raising a dragon. A diagram of energy flow of dragons is given in Figure 3. First we will analyze different states of dragons throughout their life. Then we discuss the energy spent in gaining body mass, basic metabolism, temperature maintaining, and evaporation dissipation.

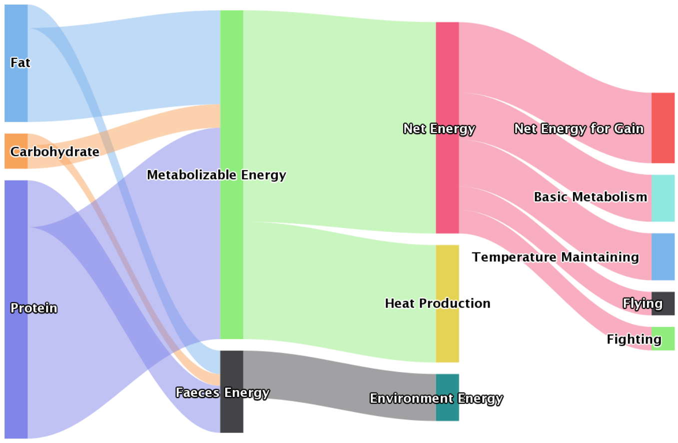


Figure 3: Diagram of energy flow of dragons. Mainly feeding on cattle, dragons absorb energy from three main nutrient substances pro rata, with part of the energy lost in faeces. The rest metabolizable energy contains heat production energy and net energy. The latter includes net energy for gain, the energy of basic metabolism, temperature maintaining, flying and fighting.

3.1 Markov Chain of Dragon-Raising

Generally, we assume dragons to be in one of the four states throughout their life, namely resting state (*State 1*), flying state (*State 2*), and fighting state (*State 3*), and a special state: dead state (*State 4*).

In additon, we postulate that the probability density function of state time obeys negative exponential distribution as

$$f(t) = \lambda e^{-\lambda t} \tag{1}$$

As can be seen in Figure 4, P_1, P_2, and P_3 are the probability of being in resting state (*State 1*), flying state (*State 2*), and fighting state (*State 3*), respectively.

Meanwhile, λ_{ij} is the transition probability from state i to state j. The state transition functions are exhibited as below

$$\begin{aligned}
\frac{\mathrm{d}P_1}{\mathrm{d}t} &= -(\lambda_{12} + \lambda_{13})P_1 + \lambda_{21}P_2 + \lambda_{31}P_3 \\
\frac{\mathrm{d}P_2}{\mathrm{d}t} &= \lambda_{12}P_1 - (\lambda_{21} + \lambda_{23})P_2 \\
\frac{\mathrm{d}P_3}{\mathrm{d}t} &= -\lambda_{13}P_1 + \lambda_{23}P_2 - \lambda_{31})P_3 \\
1 &= P_1 + P_2 + P_3
\end{aligned}$$

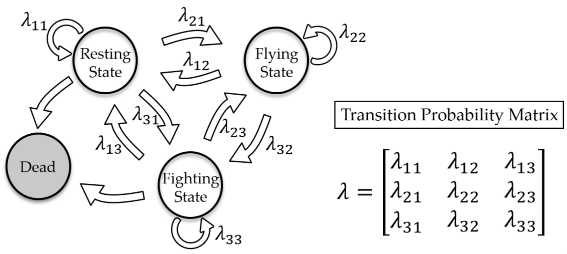


*Figure 4: Markov Chain applied to describe the sate transition of dragons. P_1, P_2, and P_3 are the probability of being in resting state (*State 1*), flying state (*State 2*), and fighting state (*State 3*), respectively. λ_{ij} is the transition probability from State i to State j.*

According to **Markov chain theorem**, the state functions possess stationary results [4]. Then the probablity of each state satisfies the equations below

$$\begin{aligned} 0 =& -(\lambda_{12} + \lambda_{13})P_1 + \lambda_{21}P_2 + \lambda_{31}P_3 \\ 0 =& \lambda_{12}P_1 - (\lambda_{21} + \lambda_{23})P_2 \\ 0 =& -\lambda_{13}P_1 + \lambda_{23}P_2 - \lambda_{31}P_3 \\ 1 =& P_1 + P_2 + P_3 \end{aligned} \tag{2}$$

Dragons have different patterns of time allocation for each state if they are being raised in the reserve or migrating. To calculate the probability of being in resting state (P_1), flying state (P_2), and fighting state (P_3), the implementation of Markov Chain Model is divided into two facets. In Section 3 we discuss the situation of being in the reserve, and the rest will be mentioned in Section 4. While dragons are being raised in the reserve, we assume that the mean time of flying, resting and fighting is respectively 4 hours, 20 hours and 1 hour, and the interval of two adjacent fighting states is 500 hours. Then we have $\frac{1}{\lambda_{12}} = 4$, $\frac{1}{\lambda_{21}} = 20$, $\frac{1}{\lambda_{23}} = 500$, $\frac{1}{\lambda_{13}} = 500$, $\frac{1}{\lambda_{12}} = 1$. Bringing these values into Equation (2), we get the final answer

$$P_1 = 0.8328, P_2 = 0.1652, P_3 = 0.002$$

3.2 Logistic Weight Prediction

In order to calculate the energy portion flowing to Net Energy for Gain (NEG), we make use of **Logistic Model** [5] to predict the weight gain of dragons, namely Logistic Weight Prediction. We assume that dragons are an integrity consisting of cells, and the weight of dragons is merely associated with the number of cells. At a given time t, the number of cells younger than r is marked as $F(r,t)$. Here we define r as the cell age, which ranges from 0 to the maximum age as r_m. Then we define $P(r,t)$ as the first order partial derivative of $F(r,t)$ with respect to r, i.e. $P(r,t) = \partial F \,/\, \partial r$ $(0 \leq r \leq r_m)$.

In our model, non-negative $P(r,t)$ stands for the age density function, a probability density function indicating the probability of cells aged r at time t. Hence, the number of cells aged $(r, r+\Delta r)$ is stated as $P(r,t)\Delta r$. After Δt, number of cells aged $(r+\Delta t, r+\Delta r+\Delta t)$ is bigger than that of Δt before due to cell proliferation during this gap time. We use $\mu(r,t)$ to denote the net proliferation rate of cells aged r, at time t. The number of extra cells is $\mu(r,t)P(r,t)\Delta r\Delta t$. Thus, this formula is given as:

$$P(r+\Delta t, t+\Delta t)\Delta r - P(r,t)\Delta r = \mu(r,t)P(r,t)\Delta r\Delta t \tag{3}$$

We make some subtle changes to Equation (3), thus getting:

$$[P(r+\Delta t, t+\Delta t) - P(r+\Delta t, t) + P(r+\Delta t, t) - P(r,t)]/\Delta t = \mu(r,t)P(r,t)$$

Let $\Delta t \to 0$, we get:

$$\frac{\partial P(r,t)}{\partial t} + \frac{\partial P(r,t)}{\partial r} = \mu(r,t)P(r,t) \tag{4}$$

The relative number of total cells $N(t)$ is expressed as

$$N(t) = \int_0^{r_m} P(r,t)\mathrm{d}r \tag{5}$$

We omit the process of formula derivation, which is exhibited in Apendix A. Defining r_0 as the maximum age of dragon cell when a dragon is born. Then we get the formula

$$N(t) = \frac{\mu_0}{\left(\mu_0 + \frac{1}{r_0}\mu_1\right)e^{-\mu_0 t} - \frac{1}{r_0}\mu_1} \tag{6}$$

3.3 Basic Metabolism

In this part, we discuss how the energy portion flowing to basic metabolism associates with the size and temperature of dragons. As stated, we regard dragons as homothermal reptiles, and consult relevant papers of biochemical kinetics and allometry based on reptiles. We define B as the basic metabolic rate of dragons, and M as the body mass. To clarify their relationship, we import **Boltzmann's factor**, namely $e^{-E_i/kT}$, where E_i is the activation energy, k is Boltzmann's constant, and T is absolute temperature [6]. With an background knowledge of the impact of body mass and temperature on metabolism, we derive the formula as

$$B \sim M^{3/4}e^{-E_i/kT} \tag{7}$$

We let $B_0 = B \,/\, M^{3/4}$, which is called *mass-normalized metabolic rate*, with an independence of body mass. Accordingly, we change Equation (7) as:

$$\ln B_0 \sim T^{-1} \tag{8}$$

From Equation (8) we can tell that $\ln B_0$ is a linear function of T^{-1}, with the slope $a = -E_i \,/\, k$. On a basis of reptile reasearch on *Science* [6], we further derive the formula:

$$\ln B_0 = -8.78T^{-1} + 26.85 \tag{9}$$

In this formula, the unit of T^-1 is $1000\,/$ degrees K, and the unit of B_0 is $W \,/\, g^{3/4}$. This result is based on big data of reptiles and will be used in our model of dragons. By noting in Equation (8) that the value of B_0 at some temperature T can be related to its value at some observed temperature T_0 by

$$\begin{aligned} \ln B_0(T) - \ln B_0(T_0) &= B_0(T_0)e^{-E_i/k(1/T-1/T_0)} \\ &= B_0(T_0)e^{-E_i(T-T_0)/kTT_0} \end{aligned} \tag{10}$$

Combining Equation (10) with Equation (7), we arrive at the expression of B as:

$$B = B_0(T_0)M^{3/4}e^{-E_i(T-T_0)/kTT_0} \tag{11}$$

3.4 Temperature Maintaining

As warm-blooded animals, it is dispensable for dragons to keep a constant body temperature so as to maintain normal life activities. The **one-dimensional heat transition model** is hired to model the maintaining of dragon body temperature [7]. We suppose that T_{in} is the temperature of dragon blood, that T_{out} is the temperature outside dragon body. There is a horny layer located between blood and outside air, whose thickness is given as L. We also define $u(x,t)$ as the temperature of location x in axis at a given time t. This is shown in Figure 5.

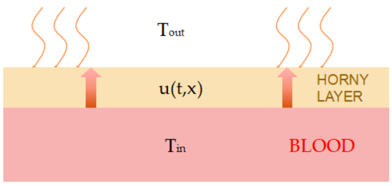


Figure 5: Diagram of a local part of dragon skin. T_{in} is the temperature of dragon blood. T_{out} is the temperature outside dragon body. The thickness of horny layer is L. We also define $u(x,t)$ as the temperature of location x in axis at a given time t.

According to **Fourier's Law of Heat Conduction**, we know that $\Phi = -\kappa\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial t} = -\frac{\partial \Phi}{\partial x}$. Here Φ stands for heat flux which means a flow of energy per unit of area per unit of time [8], and κ stands for thermal conductivity which is a measure of its ability to conduct heat [9]. In the mean time, we define that $0 \leq x \leq L, t \geq 0$. Two equations mentioned above are transformed to

$$\begin{cases} \dfrac{\partial u}{\partial t} = \kappa\dfrac{\partial^2 u}{\partial x^2} \\ u(0,t) = T_{in} \\ u(x,0) = T_{in} \end{cases} \tag{12}$$

Considering the states of dragons, the heat transition efficiency varies between resting state and flying state on account of the influence of rapidly blowing wind. Hence, we decompose this discussion into two circumstances—**Resting State** and **Flying State**.

Resting State: When dragons are resting, we think this state to meet the conditions of **Newton's Law of Cooling**. Then, we have

$$-\kappa\frac{\partial u}{\partial x}(L,t) = H\left[u(L,t) - T_{out}\right] \tag{13}$$

To simplify this model, we commands the constants $\kappa = H = L = 1$. In addition, we define a variable ν that meets the equation

$$\nu = u - T_{in} - (T_{out} - T_{in})x \tag{14}$$

We combine Equation (13) and Equation (14), and derive a series of equations: $\frac{\partial \nu}{\partial t} = \frac{\partial^2 \nu}{\partial x^2}$, $\nu(0,t) = 0$, $\nu_x + \nu|_{x=L} = 0$, and $\nu(x,0) = -(T_{out} - T_{in})x$. Therefore, we use these equatians to arrive at a result as

$$\begin{cases} u(x,t) = \sum_{n=1}^{\infty} a_n e^{-Z_n^2 t} \sin Z_n x + T_{in} + (T_{out} - T_{in})x \\ a_n = \dfrac{\int_0^1 (T_{in} - T_{out})\epsilon \sin Z_n \epsilon \mathrm{d}\epsilon}{\int_0^1 (\sin Z_n \epsilon)^2 \mathrm{d}\epsilon} \qquad (n = 1, 2, \cdots) \\ Z_n = \tan Z_n \qquad (n = 1, 2, \cdots) \end{cases} \tag{15}$$

Flying State: When dragons are flying, the heat of skin surface is quickly deprived by blowing wind. That is to say

$$u(L,t) = T_{out} \tag{16}$$

Similarly, we assume that $L = 1$. In this state, the variable ν also meets the requirement of Equation (14). Then, We combine Equation (12), Equation (14), and Equation (16), getting the result for flying state as below

$$\begin{cases} u(x,t) = \sum_{n=1}^{\infty} C_n e^{-n^2\pi^2 t} \sin n\pi x + T_{in} + (T_{out} - T_{in})x \\ C_n = 2\int_0^1 (T_{in} - T_{out})\epsilon \sin n\pi\epsilon \mathrm{d}\epsilon \qquad (n = 1, 2, \cdots) \end{cases} \tag{17}$$

3.5 Evaporation Energy Dissipation

Besides, we also consider additional energy dissipation caused by low humidity [10]. First we assume that a water molecule can evaporate from the dragon skin only when its energy is higher than the energy barrier E_p. The higher humidity is, the larger E_p is. This evaporation phenomenon is described by Figure 6.

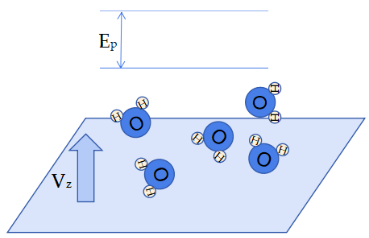


Figure 6: Diagram of evaporation taking place at the surface of dragon skin. E_p is the energy barrier that the kinetic energy of a vaporing must exceed. v_z symbolizes the vertical velocity of water molecules.

Accordingly we make a following hypothesis

$$E_p = \frac{c_t}{1-s} \tag{18}$$

In Equation (18), c_t is a constant if the temperature and atmospheric pressure of the environment are not changed, and s stands for the relative humidity. We only consider the kinetic energy of water molecules, and assume the velocity distribution of water molecules is coincident with **Maxwell distribution law of velocity**, which is shown as

$$f(v_z) = c_v e^{-\frac{mv_z^2}{2kT}} \tag{19}$$

Within Equation (19), c_v is another constant, while v_z symbolizes the vertical velocity of water molecules. According to the energy barrier assumption mentioned above, the water molecules can escape from the skin only if $\frac{1}{2}mv_z^2 > E_p$. We define n as the number density of water molecules, so the total number of the water molecules that escape from unit area ΔS in unit time Δt, namely N_{H_2O}, are exhibited as

$$N_{H_2O} = \int_{\sqrt{\frac{2E_p}{m}}}^{\infty} nv_z \Delta S \Delta t f(v_z) \mathrm{d}v_z \tag{20}$$

Meanwhile, the evaporation velocity (V_e) is calculated as

$$V_e = \frac{N_{H_2O}}{\Delta S \Delta t} \tag{21}$$

With a combination of Equation (19), Equation (20), and Equation (21), we obtain the value of V_e as

$$V_e = nc_v \frac{kT}{m} e^{-\frac{E_p}{kT}} \tag{22}$$

From Equation (18) and Equation (22), we get to know that

$$V_e \propto e^{\frac{c_t}{kT}(1-\frac{1}{1-s})} \tag{23}$$

Referring to physiology of human beings, the most comfortable relative humidity is below 70% [11]. It means that the evaportion is very weak when the relative humidity is above 70%. Accordingly we can calculate that c_t is roughly 2.5.

4 Annual Migration

4.1 Markov Chain of Annual Migration

During migration, the mean time of flying, resting and fighting is estimated to be respectively 10 hours, 14 hours and 1 hour, and the interval of two adjacent fighting states is 500 hours. Then we have $\frac{1}{\lambda_{12}} = 14$, $\frac{1}{\lambda_{21}} = 10$, $\frac{1}{\lambda_{23}} = 500$, $\frac{1}{\lambda_{12}} = 1$. Bringing these values into Equation (2), we get the probability for resting state (P_1), flyting state (P_2), and fighting state (P_3)

$$P_1 = 0.5870, P_2 = 0.4110, P_3 = 0.002$$

At last, we suppose that dragons in fighting have an 1% probability of being killed. Consequently, the probablity of death for dragons is 0.02% in both condition.

4.2 Flying Energy Dissipation

Flight is an essential part of migration, so in this part we model the energy dissipation of flying. We assume that the velocity of flying dragons is low and dragon wings are thin enough, which fits well in **the thin airfoil theory** [12]. Therefore we apply theory of **aerodynamics**. The airfoil of a dragon wing can be described by a parabola

$$y = 4f\frac{x}{c}(1 - \frac{x}{c}) \tag{24}$$

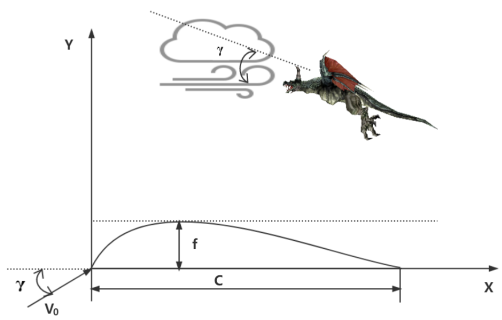


Figure 7: Diagram of the airfoil of a dragon wing. Seen from the side, the outline of a wing is described by a parabola $y = 4f\frac{x}{c}(1 - \frac{x}{c})$. The wing camber f is the maximum of y, and c stands for the chord line length of a dragon wing, stretching from the front to the back of a dragon wing airfoil. γ is hired to denote the attack angle, the intersection angle of flying direction and the chord line of a wing. v_0 is the relative air velocity during flight.

Here f is the maximum of y, which is called wing camber, and c stands for the chord line length of a dragon wing, stretching from the front to the back of a dragon wing airfoil. According to the thin airfoil theory, the lift of the dragon L is linked to the air density ρ, the relative air velocity during flight v_0, and the circulation Γ, and the wing span of the dragon l. Some important parameters are shown in Figure 7. The formula is listed as

$$L = \rho v_0 \Gamma l \tag{25}$$

$$\Gamma = \int_0^c \vartheta(\zeta)\mathrm{d}\zeta \tag{26}$$

We let $\zeta = \frac{c}{2}(1 - \cos\theta)$, and implement Fourier Expansion of ϑ.

$$\vartheta(\theta) = 2v_0(A_0 \cot\frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \sin n\theta) \tag{27}$$

After this step, we substitude Equation (27) into Equation (26), and get

$$\begin{aligned}\Gamma &= v_0 c \int_0^{\pi} \left[A_0(1+\cos\theta) + \sum_1^{\infty} A_n \sin n\theta \sin\theta\right]\mathrm{d}\theta \\ &= \pi v_0 c(A_0 + \frac{1}{2}A_1)\end{aligned} \tag{28}$$

Meanwhile, we use γ to denote the attack angle, the intersection angle of flying direction and the chord line of a wing. It is marked in Figure 7. Then we get

$$\gamma - A_0 = \frac{1}{\pi}\int_0^{\pi}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{d}\theta \tag{29}$$

$$A_n = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)cosn\theta\mathrm{d}\theta \tag{30}$$

We bring Equation (29) and (30) into Equation (24), and get the solution that $A_0 = 4f/c$, $A_n = 0$. Thus we substitde the solutions into Equation (28).

$$\Gamma = 2\pi v_0 f + \pi v_0 c\gamma \tag{31}$$

Substituding Equation (31) into Equation (25), we get

$$v_0 = \sqrt{\frac{L}{\rho\pi l(2f + c\gamma)}}$$

During the flight state of the dragons, the lift approximately equals the body mass (M) of the dragon. Known that air resistance $F = 1/2c_{\mathrm{air}}\rho S v_0^2$, where c_{air} is the coefficient of air resistance and S is the coronal section areas of the dragon, the power of flying energy dissipation P_{fly} is shown as

$$P_{fly} = \frac{1}{2\rho^2}c_{\mathrm{air}}S\left[\frac{L}{\pi l(2f + c\gamma)}\right]^{\frac{3}{2}} \tag{32}$$

5 Implementation of *DREAM* Model

5.1 Implementation of Logistic Weight Prediction

We assume that dragons will not migrate before they finish growing and that its weight is proportional to the total number of cells. Subsection 3.2 illustrates how their total cells are changed throughout their life. $N(t)$ in Equation 6 can also reflect the relative weight of dragons based on the assumptions. For simplicity, let $\mu_0 = 1.2$, $\mu_1 = -3\,/\,3250$, and $r_0 = 10$ to obtain a weight of $33.2kg$ at age 1 and a maximum weight of $130t$ as assumed. Hence, plot their relative weight with respect to t from birth to their lifespan, 220 years old, in Figure 8.

With the trend of weight change determined, energy expenditures of dragons required to keep growing could be calculated. From the curve in Figure 8, the maximum annual weight change of a dragon is $36t$ from age 7 to age 8. We take a further assumption that a dragon needs 7500 $calorie$ to gain a gram of weight. Hence, the maximum annual energy expenditures a dragon requires to maintain its growth is $1.13 \times 10^9 kJ$.

5.2 Implementation of Basic Metabolism

Energy expenditures for basic metabolism are happening all through dragons' lives. Basic metabolism includes basic biological activities to maintain vital signs

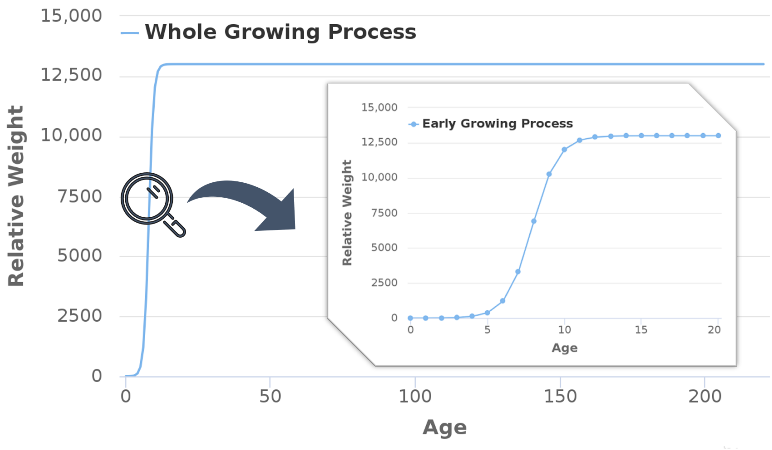


Figure 8: Relative weight of dragons throughout their whole life. When hatched, the dragons are small, roughly $10kg$ and continue to grow to a maximum of $130t$ at approximately 20 years old. Afterwards, they finish growing and maintain $130t$ for the rest of their lives.

of dragons, such as breathing, blood circulation, etc. According to assumptions, dragons are born in warm temperate regions with $293.15K$ temperature and 40% humidity. Equation 9 is adopted and let $T_0 = 293.15K$ as the reference temperature to get

$$B_0(T_0) = 0.05\ W\ /\ g^{3/4} \tag{33}$$

Substitute Equation 33 into Equation 11 and calculate the metabolic rate after dragons reaching $130t$, we obtain that

$$B = 60873\ W$$

5.3 Implementation of Temperature Maintaining

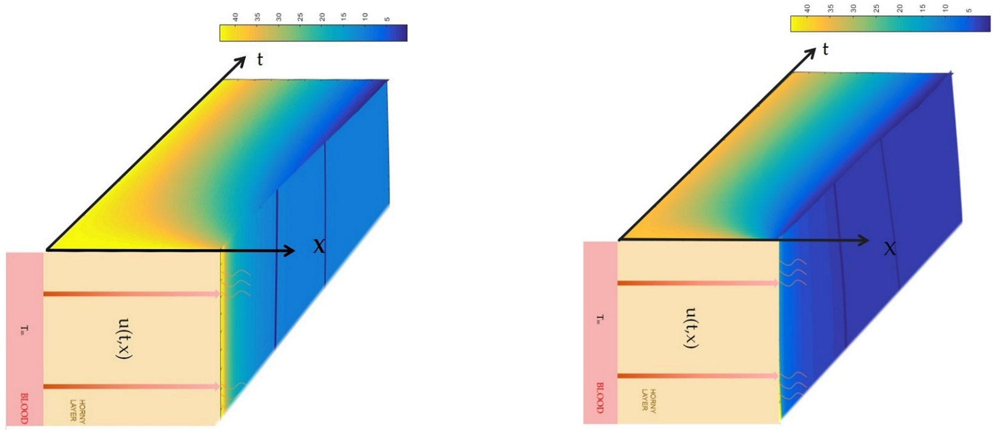
When implementing our model, we use computer to simulate the results deriving from Equation (15) and Equation (17), thus obtaining the temperature distribution in horny layer of dragon skin. Noting the color distribution difference in Figure 9 which indicates temperature distribution in horny layer of dragons, we conclude that the temperature of resting state is generally higher than that of flying state.

Moreover we move on to calculate the energy that dragons consume in maintaing body temperature. We define Φ_{super} as the superficial heat transition flux density, which is expressed by formula as

$$\Phi_{super} = \frac{\partial u(x,t)}{\partial t}, \quad x = 0 \tag{34}$$

We bring the result for $u(x,t)$ of resting state in Equation (15) and flying state in Equation (17) to Equation (34), and make a division

$$\frac{\Phi_{super,flying}}{\Phi_{super,resting}} = 1.23 \tag{35}$$



(a). Temperature distribution in horny layer of dragons in resting state

(b). Temperature distribution in horny layer of dragons in flying state

Figure 9: Simulation results of implementation of temperature maintaining model. The color is a reflection of temperature. Warm color symbolizes high temperature, and cold color symbolizes low temperature. x axis is vertical to the dragon skin surface. $u(t,x)$ is the temperature of location x in axis at a given time t in horny layer of dragons. As time (t) goes on, temperature distribution changes.

Thus it can be seen, dragons in flying state consumes 23% more energy in maintaining body temperature than in resting state, which echos the simulation result in Figure 9 and Figure 10.

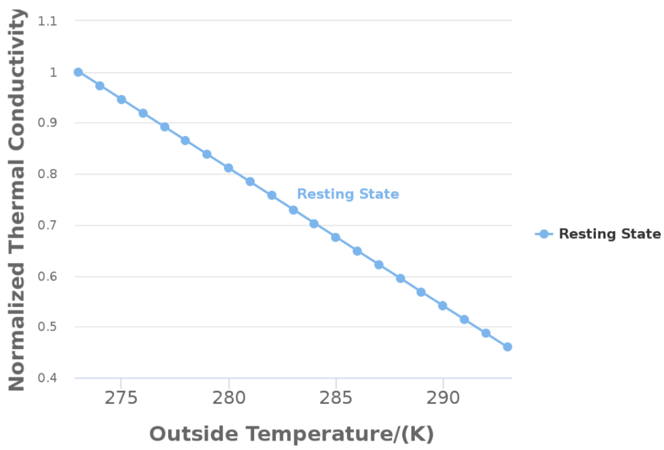


Figure 10: Normalized thermal conductivity varing with temperature. In accordance with Fourier's Law of Heat Conduction $\Phi = -K_0 \frac{\partial u}{\partial x}$, higher outside temperature causes slower heat transition.

In an effort to calculate the specific value of power consumed to maintain body temperature, we suppose that dragons have the same superficial heat transition flux density (Φ_{super}) as human beings [13]. That is to say, $\Phi_{super} = 20W/m^2$ at $20°C$. Given that a dragon weighs $130t$, and that its density is $1g/cm^3$, we estimate the superficial area of a dragon is $150m^2$. Referring to Equation 35, at last we get the power consumed to maintain body temperature in resting state ($P_{temp,rest}$) and flying state ($P_{temp,fly}$) at $20°C$ as

$$P_{temp,rest} = 3000W, P_{temp,fly} = 3900W$$

5.4 Implementation of Evaporation Energy Dissipation

We simulate the Equation (23) by means of plotting a curve line in MATLAB, as is shown in Figure 11.

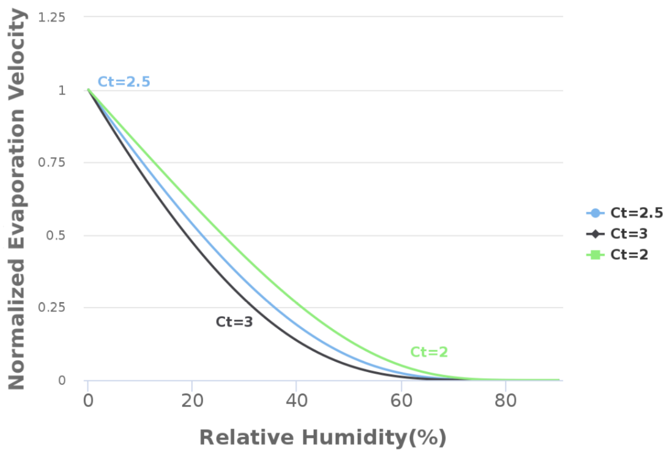


Figure 11: Normalized evaporation velocity varing with relative humidity of different c_t*. In accordance with our assumption, water evaporation of dragon skin surface weakens when relative humidity rises. When relative humidity exceeds* 70%*, evaporation almost ceases.*

It is not hard to tell from Figure 11 that, water evaporation of dragon skin surface weakens when relative humidity rises. When relative humidity exceeds 70%, evaporation almost ceases. This serves as a proof of our assumption. In an arid region where relative humidity is low, dragon skin surface evaporation rapidly increases, resulting in more energy dissipation.

To calculate the power of evaporation energy dissipation (P_{evapo}), we consult information of thermology and find that every $1g$ evaporite water takes away $244J$ energy. Analogous to human physiology, we estimate that within an atmosphere in which relative humidity is 0.4, $10g$ water evaporates in every second from dragon skin. Accordingly, we get the result of the power of evaporation energy dissipation as

$$P_{evapo} = 2440W$$

5.5 Implementation of Flying Energy Dissipation

From Equation 32, we know that the flying energy dissipation power (P_{fly}) is dependent on a series of constants. Now we give them as $G = 130t$, $f = 0.5m$, $c = 10m$, $\gamma = 0.1$, $\rho = 1.3kg/m^3$, $l = 20m$, $c_{air} = 0.2$, $S = 10m^2$, then we finally arrive at

$$P_{fly} = \frac{1}{2\rho^2} c_{\text{air}} S \left[\frac{L}{\pi l(2f + c\gamma)} \right]^{\frac{3}{2}} = 947700W$$

6 Strategy for Dragon-Raising

In this section, we will answer questions mentioned both in Problem Sheet and Problem Background, focusing on ecological impact and requirements of the dragons. That is

- Ecological impact and requirements of the dragons
- Energy expenditures of the dragons, and corresponding caloric intake requirements
- Area needed to support the three dragons
- Community size needed to offer dragons varying levels of assistance
- Impact of climate conditions on resources required to maintain and grow a dragon

In an ecological system, dragons as the top consumers in food web accelerate energy flow remarkably. After consuming a lot of food from producers and other consumers, the faeces energy finally flows to decomposer and the environment, which speeds up energy and material circulation and increases species diversity. Meanwhile, dragons are large, ferocious animals, prone to bring destructions to ecological system by means of attacking or breathing fire. Corresponding precautions are in need.

In the mean time, dragons have enormous ecological requirements, including adequate food supply, suffient living area and human assistance, which will be explained in the following contents.

In general, the dragons are raised in the reserve in temperate region, and embark on a migration every year to arid and arctic region. We decompose the strategy into two aspects—"Strategy for Dragons In the Reserve" and "Strategy for Dragons During Migration", on account of different dragon states transition patterns, climate conditions, etc.

6.1 Strategy for Dragons in the Temperate Reserve

6.1.1 Energy Expenditures

We gather all the detailed calculation results of energy expenditures in Section 5. We also assume that a dragon breathes out burning methane to produce fire for fights once in a day on average, lasting $5s$ in a time. Moreover, we assume that the area of a dragon mouth is $3m^2$, and that the velocity of methane is $20m/s$. Given the combustion heat of methane as $890.31kJ/mol$, the molar mass as $16.04g/mol$, and the density as $0.72g/L$, we know that daily energy expenditures for breathing fire of a single dragon is $1.192 \times 10^7 kJ$.

Aggregate it with all the energy expenditures derived in Section 5, we can get the overall energy required to maintain and grow a dragon in the reserve, listed in Table 2.

Sum every activity item in Table 2 together and multiply it by 3, and then we get the energy expenditures of all 3 dragons in a day:

$$E_{total} = 1.03 \times 10^8 kJ = 2.46 \times 10^7 kcal \tag{36}$$

which is also their caloric intake requirements in a day.

Table 2: Overview of daily energy expenditures for one dragon

Vital Activities	Energy Expenditures
Growth	$3.096 \times 10^6 kJ$
Basic Metabolism	$5.26 \times 10^6 kJ$
Temperature Maintaining	$2.6 \times 10^5 kJ$
Evaporation Energy Dissipation	$2.1 \times 10^5 kJ$
Flying Energy Dissipation	$1.35 \times 10^7 kJ$
Breathing Fire	$1.192 \times 10^7 kJ$

6.1.2 Living Area

With energy expenditures calculated in Equation 36, the living area needed to support the three dragons can be determined with the energy transformation in an ecosystem. We assume that the three dragons are totally supported by the energy obtained from nature without human assisstance. Globally, the average solar power that Earth receives is $1367W/m^2$. In warm temperate regions, the sunshine duration is approximately 10 hours per day so that the average solar energy on Earth in a day is $4.92 \times 10^4 kJ/m^2$.

According to ecological knowledge, the photosynthetic efficiency is about 2% and the energy transfer efficiency between different energy levels is 10%. It takes 3 energy level transitions for dragons to gain solar energy input into ecosystem. As a result, average solar energy that dragons can obtain from the ecosystem in a day is about $1kJ/m^2$. Integrated with Equation 36, we give the living area required to support the three dragons in the reserve:

$$S_{living} = \frac{1.03 \times 10^8 kJ}{1kJ/m^2} = 102.6km^2 \tag{37}$$

6.1.3 Community Size

When considering the living area above, we did not provide human assistance to dragons and let them prey on animals freely. In this subsubsection, we take into consideration various levels of assistance that can be provided to the dragons from the community. Specifically, we offer three levels of assistance to support the dragons, which are food supplies, medical care, and secure relief.

First, food from the natural ecosystem may not be enough for dragons to support their daily needs. Thus, the first level of human assistance is food supplies. As the same in the television series, the dragons mainly feed on cattle. A cattle contains $524.82kJ$ energy every hectogram[14]. Based on Equation 36 and average weight of a cattle, a dragon eats 38 cattle in total every day. On the other hand, 326 million Americans consume approximate 32 million cattle every year so that cattle consumption density is 2.68×10^{-4} per person per day. These cattle can be used to support the dragons so that we can estimate the community size to support a dragon for the first level of assistance with food supplies, which includes $142,000$ residents.

Second, dragons may get injured after fighting with each other, other animals or human beings. Hence, it is critical to support dragons with the second level of assistance, the medical care. The veterinarian density in America is about 2.02

every ten thousand people [15]. In view of the tremendous size of dragons, it may take 10 veterinarians to take care of a single dragon, thus requiring a community with $202,000$ residents to provide medical care assistance to a dragon.

Moreover, fire breathing of dragons can result in prairie or forest fires. It demands about 250 firemen to deal with secure accidents. There exists 10 firemen every ten thousand people in America [16]. Therefore, a community with $250,000$ residents are in need for the third level of assistance with secure relief.

All in all, the community sizes which are necessary to support a dragon for varying levels of assistance that can be provided to the dragons are summarized in Table 3.

Table 3: Community sizes with different levels of assistance required for a single dragon

Varying Levels of Assistance	Community Size
Food Supplies	$142,000$ residents
Medical Care	$202,000$ residents
Secure Relief	$250,000$ residents

With three dragons to provide assistance for, the community sizes in Table 3 should all be multiplied by 3.

6.2 Strategy for Dragons during Migration

As with other animals that migrate, dragons might travel to different regions of the world with very different climates. Climate conditions play an important role in the analysis of our ***DREAM*** model. Particularly, temperature and humidity are two main factors we consider in dragon migration. We compare the differences between the resources required to maintain and grow a dragon in an arid region, a warm temperate region, and an arctic region. The climate conditions in these three regions are illustrated in Table 4.

Table 4: Climate conditions in three different regions

Region	Temperature	Humidity
Arid Region	$303.15K$	5%
Warm Temperate Region	$293.15K$	40%
Arctic Region	$258.15K$	85%

Assumptions show that dragons are born in a temperate region and may migrate to an arid region or an arctic region every year. Analysis in Subsection 6.1 basically depends on the climate conditions in a warm temperate region. We specifically choose America, Africa, and Northern Europe as the representatives of these three kinds of regions, repeat the analysis process in Subsection 6.1 and derive the results of different required resources in three regions in Figure 12.

As we can see in Figure 12, a larger area and more food are required to provide for a dragon in the arctic region because of the extreme environment while less resources are needed in an arid region since more wild animals inhabit in this district [17]. Generally speaking, climate conditions make a big difference in the analysis of resources needed to provide for the dragons.

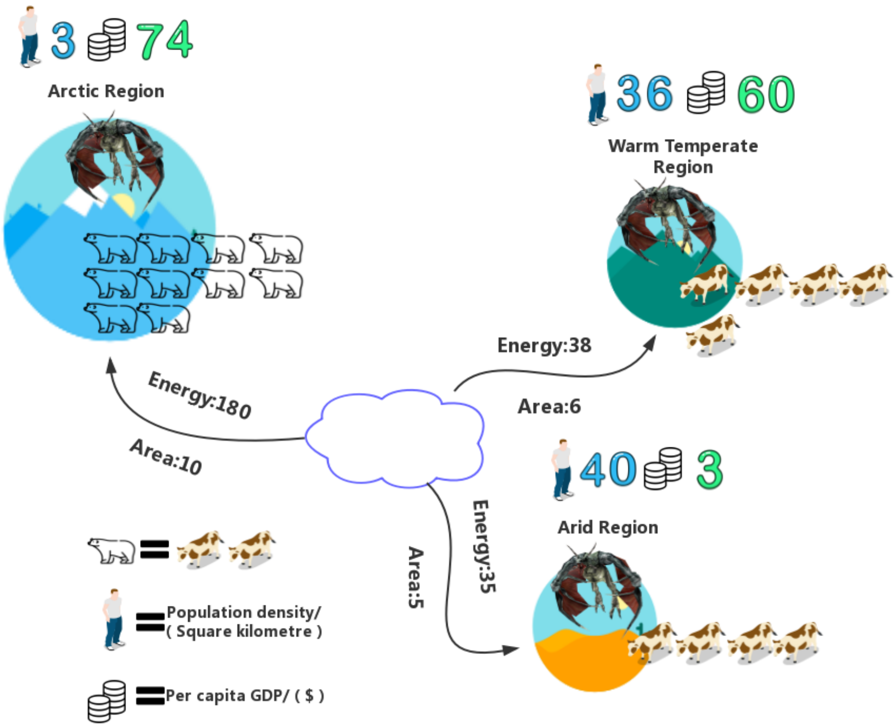


Figure 12: Different resources required to maintain and grow a dragon in a temperate region, an arid region, and an arctic region. Total energy, food, and living areas needed to provide for a dragon are demonstrated in the figure with the assumption that dragons eat polar bears instead of cattle in Northern Europe.

6.3 Model Usage Extention

Although our scheme resorts to some ideal physical models to depict living habits of fictional dragons, it has valuable and realistic applications in some biological research, especially in paleontology. Reviving mammoths is one of most popular projects nowadays. Finding cells and genes of mammoths in the layer of frozen earth brings a promising future to the study of reviving mammoths.

Due to their tremendous size and long-time extinction, it is challenging to revive mammoths in the real world. However, our ***DREAM*** model based on scientific theorems can well apply to the feeding of mammoths since we successfully model the growth and development of dragons which no one has experience in raising, just like the mammoths. ***Dragon-Raising Ecology*** model helps human beings predict the size and weight of mammoths. Because mammoths live in the ice age, research of temperature maintaining and evaporation in our model is meaningful when it comes to creating a suitable environment for mammonths as well. Considering the current climate, mammoths may migrate to different places during different seasons. Thus, our ***Annual Migration*** model can be adopted to estimate the amount of resources needed in various regions.

7 Model Analysis

7.1 Sensitivity Analysis

Dragon-Raising Ecology and Annual Migration (DREAM) model contains multiple parameters, some of which are ascertained through referring to literature, others by definition according to the need of modelling. Therefore, it is necessary to appraise the sensitivity of these given parameters to different values.

We determine the constant parameter c_t in evaporation energy dissipation model and we roughly caluclate a dragon's surface area through its weight and estimated density. Accordingly, corresponding sensitivity analysis is given below

7.1.1 Impact of Constant Parameter c_t on Evaporation Velocity

c_t is set to be 2.5 in the model based on the assumption that when the relative humidity is above 70%, the evaporation of water molecules is very weak. To observe the influence caused by different values of c_t, we choose $c_t = 2$ and $c_t = 3$ to analyse the sensitivity.

As is shown in Figure 11, we find that the evopration velocity is bigger when c_t is reduced. However, the deviation among the velocity distribution when $c_t = 2$, $c_t = 3$ and $c_t = 2.5$ is not obvious, which means that the value of c_t is not a sensitive factor of the model. And we also notice that whatever the c_t value is, evaporation velocity approaches 0 when relative humidity is close to 100%.

7.1.2 Estimation of Superficial Area of Dragon Skin

We state in the assumptions that a grown-up dragon weighs $130t$. Given this weight value and the presumptive density $1g/cm^3$, we obtain the value of dragon's volume as $130m^2$. The superficial area ratio of a sphere and a cube of the same volume is around 1.25, which is in the same order of magnitude. Accordingly we think the influence of shape on superficial area is not significant when calculating evaporation energy dissipation. Considering that the skin of a dragon may enlarge due to some wrinkles, $150m^2$ used in evaporation energy dissipation model is reasonable.

7.2 Strengths and Weaknesses

7.2.1 Strengths

1. The Dragon-Raising Ecology and Annual Migration (DREAM) Model fully considers various aspects of dragons' characteristics, behaviour, habits, diet, and interaction with the environment. We assign different weights to various factors that affects the energy expenditure and ecological requirements of dragons, in seek of an accurate simulation of raising dragons.

2. While complementing the DREAM model, the well-proved physcial theories such as Logistic Model, Fourier Law of Heat Conduction, Maxwell's Velocity Distribution Law, Thin Airfoil Theory, etc. are employed to construct the foundations, which makes this model highly credible.

3. Based on DREAM model and a great deal of associated data, we come up with a solid strategy of keeping dragons in the real world. This could be conveniently applied to many newborn creatures, especially in the near future when revival of ancient huge animals such as mammoths is possible with the development gene technology.

4. An exploitation of multi-disciplinary methods, together with a variety of explicit visulization makes it possible to vivdly present the process of giving solutions to an interesting problem.

7.2.2 Weaknesses

1. For sake of simplicity and feasibility, we leave alone some physiological factors influencing the energy dissipation and consumption of dragons.

2. We make analogy between dragons and other animals because dragons do not really exsist. Upon utilization of different constants, there are inevitable deviation of our results.

3. Due to a lack of concrete information of dragons, the DREAM model based on a lot of assumptions and estimations may not be applicable to all circumstances.

8 Conclusion

Our paper provides a detailed analysis of raising three fictional dragons in the modern world. We devise ***Dragon-Raising Ecology and Annual Migration (DREAM)*** model to describe the biological features of dragons and calculate energy expenditures and ecological requirements of dragons. Adopting multi-disciplinary methods, this model is well implemented and succeeds in obtaining energy expenditures of weight growth, basic metabolism, temperature maintaning, evaporation energy dissipation and flying energy disscipation of dragons. The probability of being in resting, flying, fighting, and death state is also derived from modeling work.

Furthermore, combining ***DREAM*** model and considerable data collected for simulation analysis, we come up with strategies for dragon-raising. When in the temperate reserve, $2.46 \times 10^7 kcal$ energy is consumed by three dragons in one day, and the same value of caloric intake is in need. A living area of $102.6 km^2$ is required to support three dragons in the reserve. Varying levels of assistance is provided by a community consisting of $142,000$, $202,000$, and $250,000$ residents, respectively for a single dragon. We also conclude from our modeling and calculation that during migration, a larger area and more food are necessary in the arctic region while less resources are needed in an arid region.

9 A Letter to Mr. George R.R. Martin

Dear Mr. George R.R. Martin:

As big fans of your masterpiece *A Song of Ice and Fire*, it is our priviledge to write to you about our perspective on how to maintain the ecological underpinning of raising three dragons in the world today. Resolved to solve this problem, we refer to a lot of data, establish our mathematical model, and simulate it with considerable ways.

Raising and migrating dragons sounds like an unrealistic dream, but we devise a ***DREAM*** model, short for ***Dragon-Raising Ecology and Annual Migration*** model, to make it come true. Set on a basis of well-proved theories and other scientific methods, this model serves well in describing the biological features of dragons and calculating energy expenditures and ecological requirements. We make an analogy between dragons, reptiles, dinosaurs, and human beings, to complete the biological analysis of dragons.

After calculating daily energy expenditures for the three dragons, we suggest that Daenerys Targaryen should provide a living area of at least $102.6km^2$ for them to obtain energy completely from the nature. Hence, Dothraki grassland could be the best place to raise the dragons.

However, Daenerys had landed on Westeros in Season 7 in the television series, so more food supplies are supposed to be provided for the dragons in Westeros. According to our calculation, about 38 cattle are needed to feed each dragon to maintain their growth and metabolism. What is more, different sizes of these three dragons are supposed to be taken into consideration when feeding them. For example, Drogon requires more food than Rhaegal and Viserion given that it is the biggest and most aggressive dragon among the three dragons.

Dragons in your book are not only living creatures, but also the weapon of wars. Daenerys rides her three dragons, starting the process of conquering the Seven Kingdoms. Therefore, it is critical to offer medical care for them after they get injured in battles. Based on our analysis, at least 10 veterinarians are needed to cure a single dragon of their injuries. As a result, we think it is more realistic to raise the three dragons near big cities with more veterinarians, like the King's Landing or the Winterfell.

Breathing fire is an important offensive pattern of these dragons. As in our calculations, it takes a dragon approximately $1.192 \times 10^7 kJ$ to breathe fire for one time. Allowing for the limited energy a dragon can obtain from the environment and human assistance of food supplies, the interval between battles for a dragon to take part in should be no less than 5 days. Otherwise, the dragon is likely to use up all its strength during a battle and cause huge damage to the ecosystem. In order to keep the dragons maintaining their fighting capacity, we believe the three dragons in the book cannot participate in the battles too frequently regardless of their tremendous power.

Due to the analysis above, we sincerely illustrate our suggestions on how to maintain the realistic ecological underpinning of story when raising these three dragons in your book in Table 5.

Aside from the suggestions above, we suppose that ecological impact on drag-

Table 5: Suggestions on how to raise a dragon in A Song of Ice and Fire

Characteristics	Suggestions
Living area	$34.2km^2$, best on Dothraki grassland
Food supplies	38 cattle needed to maintain daily growth
Medical care of injuries	live near big cities with 10 veterinarians
Frequency of battles	the interval should be no less than 5 days

ons when they move from arid regions to temperate regions and to arctic regions should also be realistic in your book. The three dragons in your book were born in arid Dothraki with Daenerys. Then they followed Daenerys to Westeros which is warm and temperate. In Season 7, Daenerys brought them beyond the Great Wall in the North to stop the Night King from attacking the human beings, where the temperature is comparable to that of the arctic regions. On the basis of our ***DREAM*** model, dragons are supposed to consume the least food in Dothraki, more in Westeros, and the most beyond the Great Wall. We demonstrate how we think the diet of the dragons should be in various regions in Figure 13.

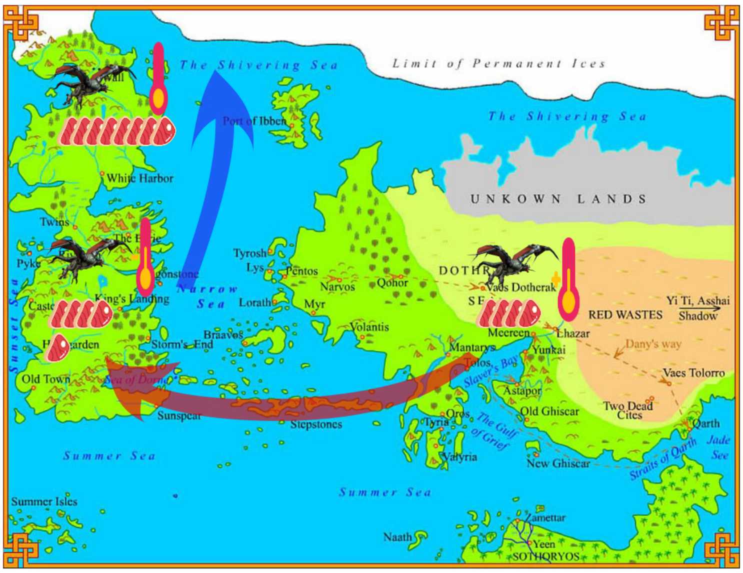


Figure 13: Different diet of dragons in various regions. In arid Dothraki where the dragons are born, they require the least energy. Then thet ate a little more in Westeros with temperate climates. For the environment beyond the Great Wall like the arctic regions, dragons need the most energy to maintain their shell temperature and basic metabolism.

In a word, we are convinced that it is critical to maintain the realistic ecological underpinning even for a fictional story. Thus, our ***DREAM*** model are appropriate to be adopted in your book to raise dragons and use dragons to fight battles in a scientific and realistic manner. The movement of dragons to various regions should be laid particular emphasis on since it plays a significant role in the growth and development of Daenerys's dragons.

We hope that you can accept our advice and produce better works in the future. Look forward to your upcoming books.

Sincerely,
Team #1919042

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Appendices

Appendix A Process of Formula Derivation of Logistic Weight Prediction

For Equation (4), defining that $P_0(r)$ is the initial density function of cells, a boundary condition is given as:

$$P(r,0) = P_0(r) \tag{38}$$

Thereafter, we look into the meaning of $\mu(r,t)$. Because net proliferation rate is decided by the number of cells to some extent, a larger $P(r,t)$ leads to less resource and more competition, resulting in a smaller proliferation rate $\mu(r,t)$. Thus, we postulate that

$$\mu(r,t) = \mu_0 + \mu_1 P(r,t) \tag{39}$$

In this linear equation, μ_0 and μ_1 are constants, especially $\mu_1 < 0$, which indicates the negative impact of $P(r,t)$ on $\mu(r,t)$. We substitude Equation (39) into Equation (4), and get the formula

$$[P(r+\Delta t, t+\Delta t) - P(r+\Delta t, t) + P(r+\Delta t, t) - P(r,t)]/\Delta t = \mu_0 P(r,t) + \mu_1 P^2(r,t) \tag{40}$$

To solve this equation, we define $\alpha = r + t$, $\beta = ar + bt$, where a and b are two arbitary constant coeffitients. Bringing them to Equation (40), we implement chain rule and get

$$2\frac{\partial P(r,t)}{\partial \alpha} + \frac{\partial P(r,t)}{\partial \beta}(a+b) = \mu_0 P(r,t) + \mu_1 P^2(r,t) \tag{41}$$

For simplification of the equation, we order that $a = 1$, $b = -1$. Then Equation (41) is changed into

$$2\frac{\partial P(r,t)}{\partial \alpha} = \mu_0 P(r,t) + \mu_1 P^2(r,t) \tag{42}$$

After separation of variables and partial fraction decomposition, we change Equation (42) into

$$\left(\frac{1}{\mu_0 P(r,t)} - \frac{\frac{\mu_1}{\mu_0^2}}{1 + \frac{\mu_1}{\mu_0} P(r,t)} \right) \partial P(r,t) = \frac{\partial \alpha}{2} \tag{43}$$

We calculate indefinite integral on both sides of the Equation (43), and displace β with $r - t$. Then we get the formula

$$\frac{1}{\mu_0} \ln \frac{P(r,t)}{1 + \frac{\mu_1}{\mu_0} P(r,t)} = \frac{\alpha}{2} + C(r-t) \tag{44}$$

Then we substitude boundary condition Equation (38) into Equation (44), thus getting

$$C(r) = \frac{1}{\mu_0} \ln \frac{P_0(r)}{1 + \frac{\mu_1}{\mu_0} P_0(r)} - \frac{r}{2} \tag{45}$$

If r is replaced by $(r - t)$, then we get

$$C(r-t) = \frac{1}{\mu_0} \ln \frac{P_0(r-t)}{1 + \frac{\mu_1}{\mu_0} P_0(r-t)} - \frac{r-t}{2} \tag{46}$$

With Equation (46) combined with Equation (44), we get the solution of $P(r,t)$

$$P(r,t) = \frac{\mu_0 e^{\mu_0 t} P_0(r-t)}{\mu_0 + \mu_1 (1 - e^{\mu_0 t}) P_0(r-t)} \tag{47}$$

In Equation (47), $P_0(r)$ is the initial density function of cells as stated before. For the sake of simplicity, we think of $P_0(r)$ as a uniform distribution

$$P_0(r) = \begin{cases} \frac{1}{r_0}, & 0 \le r \le r_0 \\ 0, & \text{otherwise} \end{cases} \tag{48}$$

Here r_0 is the maximum age of dragon cell when a dragon is born. We aggregate Equation (48), Equation (47) and Equation (5), then we get the solution of $N(t)$ in Equation (6)

Appendix B Logistic Model to Predict Weight

```
clear;clc;

%Define variable of r,t
syms r t

%Define parameters
mu0=1.2;
mu1=-3/3250;
aver=100;
delta=20;
p=1/10;

%Defination of probability function
P=mu0*exp(mu0*t)*p/(mu0+mu1*(1-exp(mu0*t))*p);
N=int(P,r,0,1/p);

%Initialize the vector N1 with zeros
N1=zeros(1,221);
for t=0:220
    N1(1,t+1)=double(subs(N));
end

%Plot the function
t1=0:220;
plot(t1,N1);

fid=fopen('data.txt','w');
for i=1:21
    fprintf(fid,'%6.2f,\n',N1(1,i));
end
fclose(fid);
```

Appendix C Metabolic Rate with Weight and Temperature

```
clear;clc;

%Value of metabolism energy
M=10000:1000:130000000;

%Combine the metabolism and mass
B=4320*M.^(3/4)/1000;

%Loop through temperature from 258 to 303
T=258:303;

%Calculate metabolism with temperature
B1=4320*(130000000)^(3/4)/1000*exp(-6956.5*(T-293.15)./T/293.15);

%Plot figure
plot(M,B);
figure(2)
plot(T,B1);
```

Appendix D Temperature Distribution in Body Surface

```
% Initialize the list zn
z=[];
n=1;

%Initialize the parameters
pi=3.1415926
pimax=3.1415927

%The step of simulation
step=0.0000001;
temp=step+pi/2;

%Consider only first 20 zn
while n<20
    z=[z,n*pi]
    n=n+1;
end

b=0;
a=37;
n=1;
alpha=[];

%Calculate the coincident alpha parameter
while n<20
    tt=z(n);
    ac1=@(x)2.*(a-b).*x.*sin(tt*x);
    temp=quadgk(ac1,0,1);
    alpha=[alpha,temp];
    n=n+1;
end

alpha
v=[];
step2=0.01

%Calculate the v map of time and length
for l=0:step2:1
    u=[];
    for t=0:step2:1
        temp=0;
        for i=1:19
            temp=temp+alpha(i)*exp(-(z(i)*z(i))*t)*sin(z(i)*l);
        end
        u=[u,temp+a-(a-b)*l];
    end
    v=[v;u];
end

v
w=zeros(101,101);

%Calculate the temperate at the boundary of skin and air
```

```
for t=0:step2:1
    fix(t*(1/step2)+1);
    v(fix(1/step2+1),fix(t*(1/step2)+1));
    w(:,fix(t*(1/step2)+1))=v(fix(1/step2-5),fix(t*(1/step2)+1));
end

%Plot figure
imagesc([0,10],[0,10],w),colorbar
xlabel('Time(s)')
ylabel('Length(cm)')
sum(v(1,:))-sum(v(2,:))
```